Spin transverse separation in a two-dimensional electron-gas using an external magnetic field with a topological chirality

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We propose a two-dimensional electron-gas (2DEG) system in which an external magnetic field with a small chirality is applied to provide a topological spin gauge field that separates conduction electrons of opposite spins in the transverse direction. Additionally, the vertical electric field in the 2DEG, together with spin-orbit coupling, produces a SU(2) gauge field which reinforces or opposes the effect of the spin gauge. The system thus provides a tunable spin separation effect, where an applied gate voltage on the 2DEG can be used to modulate the transverse spin current. As this method leads to the enhancement or cancellation of spin separation due to the intrinsic spin-orbit coupling effect only, it may naturally distinguish the extrinsic effect from the intrinsic one.

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In recent years, there have been various spintronic propositions with respect to device functionality, foremost of which is the Datta-Das[1] transistor that utilizes the Rashba[2–4] spin-orbit (SO) effects to induce spin precession across the two-dimensional electron-gas (2DEG) conduction channel. Subsequent experiments[5,6] have confirmed the working principles of such devices but failed to observe a large conductance modulation. In parallel to these developments, Majumdar[7] and others[8,9] have shown the theoretical possibility of using external delta (δ) magnetic fields to induce spin polarization. However, such devices are difficult to implement because they require spatially concentrated magnetic fields in order to approximate the δ-function distribution.

Recently, attention has been shifted to utilizing the SO coupling in the presence of longitudinal electric field to generate a pure spin transport in the transverse direction of the 2DEG commonly known as the spin-Hall effect (SHE). Inoue *et al.*[10] showed that the spin-Hall conductivity vanishes when vertex corrections are introduced to model the effects of impurity scattering. The suppression effect is, however, not generally true; for instance, in bulk *p*-type semiconductors[11] and two-dimensional-hole gases,[12] the SHE persists in the presence of impurities. The SHE based on Sinova *et al.*[13] falls under the more general topic of transverse spin-dependent transport. Sinova *et al.*[13] showed that in the ballistic limit, accelerated electrons in heterostructures with Rashba spin-orbit coupling (SOC) with a net left transverse velocity precess in the opposite direction to those traveling to the right, resulting in a transverse spin separation. Such spin-Hall effect vanishes in the diffusive limit due to impurity scattering causing a steady cancellation[14] of the required acceleration. This effect is consistent with the prediction of Inoue *et al.*[10] mentioned earlier. In the context of non-Abelian gauge fields[15,16] due to the spin-orbital effect, a similar form of spin transverse separation is predicted in Ref. 17 in which a vertically spin-polarized current in a ballistic 2DEG system with Rashba SOC experiences spin transverse forces. It is, however, unclear at this point whether the two mechanisms are at all related, apart from the fact that both predict some form of transverse spin separation. On the other hand, in Refs. 18–21, the out-of-plane spin polarization along the edges of ballistic 2D spin-Hall systems was studied. The out-of-plane spin components there are driven by edge precession effects,[18] and the resulting edge spin accumulation can constitute an observable signature of the spin-Hall effect in 2DEG systems. This is in contrast to the SHE of Ref. 13, where the spin precession of carriers which are accelerated by an external electric field in the presence of Rashba SOC pushes the in-plane spins in the out-of-plane direction and generates the transverse separation of z-polarized spins. The above-mentioned works collectively study the so-called intrinsic version of the SHE which arises from the SOC in the band structure of the system. In contrast, the extrinsic SHE is driven by spin-dependent scattering mechanisms of carriers with impurities, namely, via the skew-scattering and side jump mechanisms.[22]

In this paper, we follow the elucidation of transverse spin separation in the context of spin transverse force, in which to achieve a significant transverse spin current,[17] one requires (i) spins to be polarized in the vertical direction and (ii) a strong SU(2) gauge field, resulting from the large electric $\boldsymbol{E}$ field perpendicular to the 2DEG plane (the Rashba SOC). However, the requisite vertically spin-polarized state is not an eigenstate of the system, since the vertical $\boldsymbol{E}$ field produces a relativistic magnetic field in the in-plane direction. This leads to two possible effects, both detrimental to the spin current: (i) the large $\boldsymbol{E}$ field will hasten the relaxation of the initial vertical spins to the in-plane direction, thereby suppressing the SU(2) transverse force or, alternatively, (ii) for channel lengths longer than the spin coherence length, the spin vector will precess about the in-plane relativistic magnetic field, causing a *Zitterbewegung*-like motion and resulting in zero net transverse spin current.[17] Thus to prevent either the spin relaxation or precession of the vertical spin

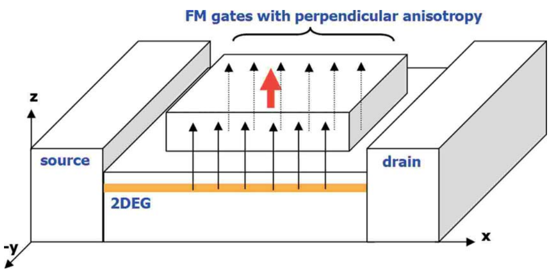
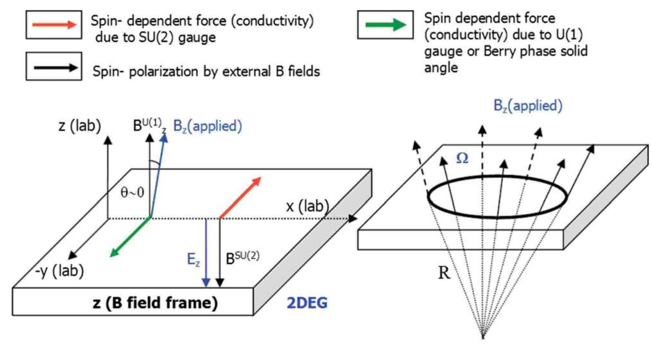


FIG. 1. (Color online) Schematic illustration of a lateral device with ferromagnetic gates that utilize the gauge fields to realize transverse spin separation. The dotted arrows represent the gate magnetization. The schematically vertical $\boldsymbol{B}$ fields represent a two-dimensional distribution of $\boldsymbol{B}$ fields with a small chiral angle as shown in Fig. 2.



(a) (b)

FIG. 2. (Color online) (a) Schematic illustration of the forces in opposite directions with proper setting of the field directions. The transverse spin current polarized along z and flowing along the x direction can be modulated or reversed by tuning and reversing the E_z field. (b) Schematic illustration of applied field distribution with net chirality characterized by solid angle Ω.

state, one needs a very short device channel and minimal scattering, which would be a difficult task to implement in practice. Based on the above discussions, we thus propose a 2DEG-based system which removes the competition between the SU(2) force and the spin relaxation or precession due to the strong vertical $\boldsymbol{E}$ field in the 2DEG and also provides insights into possible experimental implementation. The device utilizes an external magnetic field $\boldsymbol{B}$ applied perpendicular to the 2DEG plane together with the Rashba SO coupling. The use of a magnetic field to lock the spins, thereby suppressing spin-relaxation and spin-precession effects, has been studied previously in the context of the relative contributions of the intrinsic and extrinsic SHE in 2DEG systems.[22] The $\boldsymbol{B}$ fields in the present system are applied by means of ferromagnetic gate stripes[23,24] deposited on top of a high-electron-mobility-transistor (HEMT) heterostructure device, as shown in Fig. 1.

The $\boldsymbol{B}$ field applied here should be sufficiently strong relative to the in-plane relativistic magnetic field so as to ensure that the two spin eigenstates point in the out-of-plane direction, ideally close to parallel and antiparallel to the z axis. However, we assume the $\boldsymbol{B}$ field has a weak influence on spin polarization of current across the device, i.e., we assume the conduction electrons are almost evenly split between the two eigenstates. In summary it is required that $E_{\text{spin orbit}} < E_{\text{Zeeman}} < E_{\text{thermal}}$ (here E denotes energy). In a typical III-V 2DEG heterostructure, e.g., InAs/InGaAs with a Rashba constant of $\alpha = 10^{-12}$ eV m and moderately low doping density of $n = 10^{13}$ m^{-2}, numerical evaluation shows that the $\boldsymbol{B}$ field of strength 0.1 T is sufficient to achieve $E_{\text{Zeeman}} > 5\,E_{\text{SOC}}$ to ensure adiabaticity. As E_{thermal} is large at 25 meV, the above requirement is clearly satisfied.

A spatial nonuniformity is superimposed onto the uniform vertical $\boldsymbol{B}$ field in order to introduce a finite chirality [see Fig. 2(b)]. Thus, the applied $\boldsymbol{B}$ field serves a twofold purpose: (1) to counteract suppression of the SU(2) force due to spin relaxation to the in-plane direction and (2) to provide a topological spin separation, arising from the spatial nonuniformity of the field. Therefore, spin transverse separation is achieved by virtue of the SU(2) gauge, as well as the $U(1)\otimes U(1)$ topological gauge[25–27] related to the chirality of the external $\boldsymbol{B}$ field. Transistor action can be achieved via the quantum-mechanical force due to the former SU(2) gauge, which can be tuned via a gate bias. Figure 2 shows the schematic illustration of the various forces. We note that the argument of force is heuristic. It will be the equation of motion (EOM) which provides the clear affirmative to spin transverse separation. In this design, the external $\boldsymbol{B}$ fields are required to ensure that electron spins align adiabatically along the field direction and do not directly contribute to the transverse separation of spins. This is because external $\boldsymbol{B}$ fields could only give rise to "Lorentz force" that is not spin dependent and will result in charge separation only, i.e., the ordinary charge Hall effect. The $\boldsymbol{B}$ field we need is only of low strength (<0.1 T), i.e., sufficiently high to ensure adiabaticity. This value can be further lowered with engineering optimization and proper choice of materials which will not be the focus of this paper. The strength of the effective magnetic field (4 mT), which is needed for the transverse spin separation, depends on the chirality of the real $\boldsymbol{B}$ field but not its strength. The above reasoning rules out the relevance of the quantum Hall effect[28] to our present studies.

To provide a theoretical description of the device, we write the Hamiltonian of the system as

$$H = \frac{p_x^2}{2m} + \frac{(p_y + eA_y^B)^2}{2m} + \frac{eg\hbar}{4m}\boldsymbol{\sigma}\cdot\mathbf{B} + \frac{\hbar e}{4m^2c^2}\boldsymbol{\sigma}\cdot[(\mathbf{p} + e\mathbf{A}^{\mathbf{B}}) \times \mathbf{E}], \tag{1}$$

where $\boldsymbol{\sigma}$ is the vector of Pauli-spin matrices, $\mathbf{A}^{\mathbf{B}} = (0, A_y^B, 0)$ is the Landau gauge associated with the external $\boldsymbol{B}$ field, and $g\hbar/4m$ is the Zeeman splitting strength. Equation (1) can be transformed to

$$H = \frac{1}{2m}\left(p_x + \left[\frac{\hbar e}{4mc^2}\sigma_i E_j \varepsilon_{ijx}\right]\right)^2 + \frac{1}{2m}\left(p_y + e\left[A_y^B + \frac{\hbar}{4mc^2}\sigma_i E_j \varepsilon_{ijy}\right]\right)^2 + \frac{eg\hbar}{4m}\boldsymbol{\sigma}\cdot\mathbf{B}, \tag{2}$$

after ignoring the higher order terms. Performing a unitary transformation of Eq. (2) with the rotation operator U, so that all local $\boldsymbol{B}$ field orientations are transformed to the z axis, we obtain

$$H' = UHU^{+} = \sum_{k=x,y,z} \frac{1}{2m}\left(p_k + e\left[A_k^B + \frac{\hbar}{4mc^2}U\sigma_i E_j \varepsilon_{ijk}U^{+} + \frac{i\hbar}{e}U\partial_k U^{+}\right]\right)^2 + \frac{eg\hbar}{4m}\sigma_z B_z. \tag{3}$$

Under the adiabatic condition, the 2×2 matrix $U\partial_k U^{+}$ becomes diagonal, hereafter represented by matrix A_k^M (whose elements are the monopole potential). Taking note that the electric field is vertical to the 2DEG plane in the Rashba system, the transformation $U(\sigma_i E_j \varepsilon_{ijk})U^{+}$ is equivalent to rotating the laboratory z axis to the $\boldsymbol{B}$ field axis (see Fig. 2). Thus, the gauge fields comprise the $A^{\mathrm{SU}(2)}$ term from the spin-orbital effects, and the topological term arising from the net chirality of the $\boldsymbol{B}$ field

$$A_k^M = \begin{pmatrix} a_k & 0 \\ 0 & -a_k \end{pmatrix},$$

where a_k is the $U(1)$ monopole potential. In the presence of both $U(1)\otimes U(1)$ and SU(2) gauge fields, an electron propagating back to its initial spin state through various closed paths results in the electrons acquiring a phase equal to the solid angle due to the $\boldsymbol{B}$ field's chirality. This statement explains the connection between the gauge-related "force" and the Berry's phase because the "force" arises from the curvature of the gauge field while the Berry's phase arises from the integral of the gauge field over a closed path. However, what is really needed in the device is the curvature of the gauge field to result in the spin-dependent Lorentz "force" and not the Berry's phase. In other words, the device can function regardless of whether Berry's phase can be resulted. The diagonal components of the monopole gauge matrix $(\pm a_k)$ can be understood by the path-integral method, $\psi(x,t)=\int G(xt,x_0t_0)\psi(x_0,t_0)dx_0$ (Refs. 29 and 30) for spin parallel or antiparallel to the $\boldsymbol{B}$ field, where $G(xt,x_0t_0)$ is the propagator between times t_0 and t; x and t are, respectively, x_{N+1} and t_{N+1}. Explicitly, the evolution is described by

$$\psi(x,t) = \int \left[\left(\frac{m}{2\pi i\hbar\Delta t}\right)^{\frac{n+1}{2}} \gamma_n \cdots \gamma_0 dx_n \cdots dx_1\right]\psi(x_0,t_0)dx_0,$$
$$\text{where } \gamma_n = \exp\left(\left[\frac{m}{2}\left(\frac{x_{n+1}-x_n}{\Delta t}\right)^2 - V(x_n)\right]\frac{i\Delta t}{\hbar}\right)$$
$$\text{and } V(x_n) \tag{4}$$

is the local potential. Equation (4) can be written in a simple form,

$$\psi(x,t) = \int \left[\left(\frac{m}{2\pi i\hbar\Delta t}\right)^{(n+1)/2} e^{iS(t)/\hbar}dx_n \cdots dx_1 dx_0\right]\psi(x_0,t_0), \tag{5}$$

where $S(t)=\frac{i}{\hbar}\int_0^T \frac{m}{2}(\frac{dx}{dt})^2 - V(x_n)dt$ is the action of the system. In the dynamic spinor system, S corresponds to the action $S[n(t)]=\frac{\hbar}{2}\int_0^t iz^{+}\partial_t z dt$ where $n(t)$ refers to the spinor vector at time t.[29] An expansion of the action leads to

$$S = \pm\frac{\hbar}{2}\int_0^t iz^{+}\partial_t z dt = \pm\frac{\hbar}{2}\int_0^t (1-\cos\theta)\frac{\partial\phi}{\partial t}dt, \tag{6}$$

for spin parallel (+) and antiparallel (−) to field, respectively. Considering the evolution over a short-time interval Δt, the above integration leads to

$$\int \pm\mathbf{a}\cdot d\mathbf{r} = \pm\int \frac{\hbar}{2e}(1-\cos\theta)\nabla_r\phi\cdot d\mathbf{r}. \tag{7}$$

The effect of the gauge fields on the electron motion, for spin assuming out-of-plane eigenstates under the applied $\boldsymbol{B}$ field, can be reduced to examining the individual effect of $A^{\mathrm{SU}(2)}$ and A_k^M. The partial spin polarization induced by the external $\boldsymbol{B}$ field can be described by $|\psi\rangle\langle\psi| = c^{\uparrow}|\psi_{\uparrow}\rangle\langle\psi_{\uparrow}| + c^{\downarrow}|\psi_{\downarrow}\rangle\langle\psi_{\downarrow}|$, where $c^{\uparrow}$ and $c^{\downarrow}$ are some functions of temperature or other material parameters and with $c^{\uparrow}+c^{\downarrow}=1$. In our system, a weak spin polarization has been assumed so that $c^{\uparrow}\approx c^{\downarrow}=0.5$. As $\boldsymbol{A^B}$ acts on both up and down spins in the same transverse direction, a weak vertical polarization implies that $\boldsymbol{A^B}$ would not contribute to the transverse spin separation. We will thus focus on the effective magnetic fields generally prescribed by the Yang Mill field tensor of

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \frac{ie}{\hbar}[A_\nu, A_\mu]. \tag{8}$$

Focusing on the spin-dependent part of the curvature and using the relation $[U\lambda_\nu U^{+}, U\lambda_\mu U^{+}] = U[\lambda_\nu,\lambda_\mu]U^{+}$ where $\lambda_{\nu,\mu}$ is an arbitrary vector component, an effective field for the SU(2) gauge field can be obtained:

$$\mathbf{b}_{\mathbf{Z}}^{\mathrm{SU}(2)} = \frac{\hbar e}{8m^2c^4}(U\boldsymbol{\sigma}U^{+}\cdot\mathbf{E})E_z\mathbf{n}_{\mathbf{z}}, \tag{9}$$

where $\mathbf{n}_{\mathbf{Z}}$ is a vertical unit vector. The non-Abelian nature of the gauge arises from the noncommutativity of the SU(2) spin algebra. Noting that $\boldsymbol{E}$ is vertical in the 2DEG system, we find that for an electron traveling along the laboratory x axis with spin parallel to the $\boldsymbol{B}$ field, the transverse force operator is

$$\hat{F}_y^{\mathrm{SU}(2)}\mathbf{n}_{\mathbf{y}} = \frac{\hbar e^2}{8m^2c^4}(\sigma_z\cos\theta)\hat{v}_x E_z^2\mathbf{n}_{\mathbf{y}}. \tag{10}$$

Hereafter, we will denote operators with a hat (ˆ). Note that the local $\boldsymbol{B}$-field configuration has a net chirality with a small solid angle Ω, hence the angle θ. The approximation $U(\sigma_i E_j\varepsilon_{ijk})U^{+} = (E_j U\sigma_i U^{+})\varepsilon_{ijk} \approx (\sigma_i E_j)\varepsilon_{ijk}$ which holds for small θ would lead to a force operator of $\hat{F}_y^{\mathrm{SU}(2)}\mathbf{n}_{\mathbf{y}} = (\frac{\hbar e^2}{8m^2c^4})\sigma_z\hat{v}_x E_z^2\mathbf{n}_{\mathbf{y}}$ which is consistent with Eq. (10). From Eq. (8), the effective magnetic field due to the $U(1)\otimes U(1)$ gauge was obtained from its curvature, which is Abelian: $\mathbf{b}_{\mathbf{z}}^{\mathbf{M}} = \frac{\hbar}{2eR^2}\sigma_z\mathbf{n}_{\mathbf{z}}$, where R is the monopole radius [see Fig. 2(b)]. The force operator related to this field is of Lorentz type,

$$\hat{F}_y^M \mathbf{n_y} = -\frac{\hbar}{2R^2}\sigma_z \hat{v}_x \mathbf{n_y}. \quad (11)$$

The spin-dependent force of Eq. (10) provides a heuristic indication of possible spin separation. In the following, we will provide a more definitive picture of spin separation by deriving the position operator in the Heisenberg picture in the two-dimensional Rashba semiconductor system. The position and velocity operators method has been used previously in the literature[16,31] to derive the equations of motion in semiconductor systems. In fact a force operator can be related to the time derivative of the velocity operator in the nonrelativistic limit. This force operator is, however, different from the curvature force operator of Eq. (10). Nevertheless, both force operators should yield the same EOM by taking their expectations

$$\hat{y}(t) = e^{i\hat{H}t/\hbar}\hat{y}(0)e^{-i\hat{H}t/\hbar} = \sum_{n=0}^{\infty}\left(\frac{\tau^n}{n!}\right)[\hat{H},[\hat{H}\cdots[\hat{H},\hat{y}]\cdots]], \quad (12)$$

where $\tau = it/\hbar$, and $\hat{H}$ is the Hamiltonian of Eq. (3). Using the Baker-Campbell-Hausdorff relation and summing the odd and even terms carefully, the position operator is

$$\hat{y}(t) = \hat{y}(0) + \frac{\alpha\sigma_y}{2B_z} + \frac{\hat{p}_y t}{m} - \left(\frac{\alpha^2\hat{p}_x\sigma_z}{\hbar^3} - \frac{eB_z\hat{p}_x}{m^2} - \frac{eb_z^M\hat{p}_x\sigma_z}{m^2}\right)t^2 - \frac{\alpha\sigma_y}{2B_z}\cos\left(\frac{2tB_z}{\hbar}\right) - \frac{\alpha\sigma_z}{2B_z}\sin\left(\frac{2tB_z}{\hbar}\right), \quad (13)$$

where α is the Rashba constant. To obtain a measurable spin transverse separation,[32,33] we represent the electron's probability amplitude with a Gaussian wave packet of width d in k space prepared in the spin-up state, i.e.,

$$\psi_{\uparrow,k}(\mathbf{r},t) = \frac{d}{2\pi\sqrt{\pi}}\int d^2k e^{-1/2d^2(\mathbf{k}-\mathbf{k}_0^2)} e^{i\mathbf{k}\cdot\mathbf{r}}\begin{pmatrix}1\\0\end{pmatrix}. \quad (14)$$

Assuming that the expectation value of the transverse wave vector is zero, i.e., $\langle\psi_{\uparrow,k}|\hat{k}_y|\psi_{\uparrow,k}\rangle = k_{y0} = 0$, the spin-dependent average transverse separation of electrons is

$$\langle\psi_{\uparrow\downarrow}|\hat{y}(t)|\psi_{\uparrow\downarrow}\rangle = y(0) + \left(\frac{eB_z t^2}{m^2} \pm \frac{b_z^M t^2}{m^2} \mp \frac{\alpha^2 t^2}{\hbar^3}\right)\hbar k_{x0}. \quad (15)$$

Note that $\uparrow/\downarrow$ corresponds to up or down spin, respectively, and $\hat{y}(t)$ is the full transverse position operator of Eq. (13). Equation (15) shows spin separation in the transverse direction. In fact spin separation of Eq. (15) arises from the spin separation part of Eq. (13), i.e., $\hat{y}_s(t) = -\frac{\alpha^2 t^2}{\hbar^3}\hat{p}_x\sigma_z + \frac{eb_z^M t^2}{m^2}\hat{p}_x\sigma_z$. Recalling the curvature force operator of Eq. (10), it can be quickly shown that the position operator that arises directly from it is $\hat{y}(t) = \frac{\hbar e^2}{16m^3c^4}(\sigma_z\cos\theta)\hat{v}_x E_z^2 t^2$. This expression is exactly the same as the spin separation part of Eq. (13) upon converting to the Rashba constant and removing the chirality strength of $\cos\theta$. We have thus shown that the curvature force operator can give rise to the spin separation operator that produces the spin separation of Eq. (15) for electrons prepared in the Gaussian wave-packet initial spin-up state.

In the following, we will study the tunability of spin separation. For simplicity, we will study the expectation of the force operators instead of the spin separation since it has been shown to be directly linked to the transverse spin separation. Inspection of all the transverse force expressions derived earlier shows that both are spin dependent and point in opposite directions for the two spin orientations. In addition, the effective magnetic-field directions suggest that the two forces can be designed to reinforce or to oppose one another, thus enhancing or canceling any form of spin separation. We consider a device where the ***B*** and ***E*** fields are along the $+z$ and $-z$ directions, respectively, and their magnitudes are such that the resultant spin separation effects completely cancel one another. As one modulates the asymmetric ***E*** field by varying the gate potential,[5,6] the transverse $\langle\hat{F}^{SU(2)}\rangle$ force from the SO interactions declines in strength, resulting in spin separation favoring one transverse side. The device can thus be used to turn on and off the spin transverse accumulation. One can derive the expectations of the transverse force operators of the spin-orbit-induced curvature for the Gaussian wave packet defined in Eq. (14) to be

$$\begin{aligned}\langle\psi_{\uparrow,k}|\hat{F}^{SU(2)}\mathbf{n_y}|\psi_{\uparrow,k}\rangle &= \frac{\hbar e^2 E_z^2\cos\theta}{8m^2c^4}\langle\psi_{\uparrow,k}|\sigma_z\hat{v}_x\mathbf{n_y}|\psi_{\uparrow,k}\rangle\\ &= \frac{\hbar^2 e^2 E_z^2\cos\theta}{8m^3c^4}\langle\psi_k|\hat{k}_x|\psi_k\rangle\mathbf{n_y}\\ &= \frac{\hbar^2 e^2 E_z^2\cos\theta}{8m^3c^4}k_{x0}\mathbf{n_y},\end{aligned} \quad (16)$$

where

$$|\psi_{\uparrow,k}\rangle = |\psi_k\rangle\begin{pmatrix}1\\0\end{pmatrix}$$

and, likewise, for the monopole curvature induced by the chiral magnetic field to be

$$\langle\psi_{\uparrow,k}|\hat{F}_y^M\mathbf{n_y}|\psi_{\uparrow,k}\rangle = -\frac{\hbar}{2R^2}\langle\psi_{\uparrow,k}|\sigma_z\hat{v}_x\mathbf{n_y}|\psi_{\uparrow,k}\rangle = -\frac{\hbar^2}{2mR^2}k_{x0}\mathbf{n_y}. \quad (17)$$

To illustrate the effects in a practical system, we consider a 2DEG formed in an InAs/InGaAs heterostructure[5,6] with material parameters: effective electron mass $m^* = 0.05m_0$, charge density $n_e = 10^{13}$ m^{-2}, and Fermi wave vector $k_F = 7.9 \times 10^6$ m^{-1}. The external ***B*** field is applied on the 2DEG plane such that it inscribes a circular distribution of radius $r = 5$ nm, with a field orientation θ from the vertical and a radially outward azimuthal direction [see Fig. 2(b)]. We consider a nonequilibrium condition due to the application of a longitudinal electric field. The typical Rashba constant $\alpha = 10^{-12}$ eV m in 2DEG translates to an effective ***E*** field strength $E_z = 0.67 \times 10^{11}$ V m^{-1}. The application of a gate voltage can affect the band-bending within the 2DEG heterostructure, thus modifying the value of α and hence E_z of up to $\sim 50\%$.[5,6] Adjusting the gate voltage to vary the E_z field, we found (Fig. 3) that the total average transverse force can

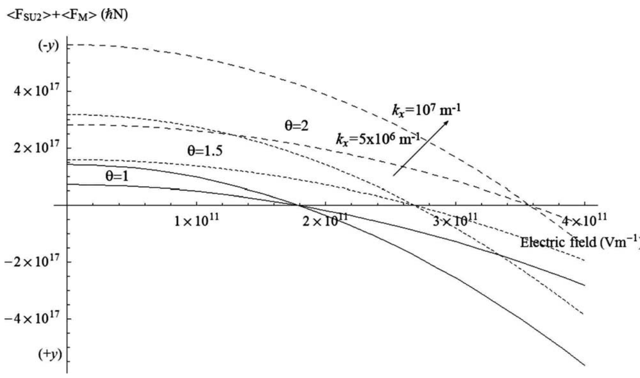


FIG. 3. The sum of the expectation value of the two forces [Eqs. (16) and (17)] evaluated for an injected Gaussian wave-packet state for three magnetic-field configurations θ (degree), in an InAs/InGaAs 2DEG. The dependence on the average wave vector k_x is also shown for each configuration; this quantity reflects drift velocities in such systems in the presence of a longitudinal electric field. As the effective electric field perpendicular to the 2DEG is increased, the component of the force due to spin-orbit coupling becomes significant compared to the constant topological force. At a critical $\boldsymbol{E}$ field value, the two forces cancel one another completely, switching off the transverse spin current.

be modulated to switch on and off spin separation. We illustrated this for three $\boldsymbol{B}$ field orientations of $\theta = 1°, 1.5°, 2°$ (see Fig. 2 for θ). In each case the $\boldsymbol{E}$ field-dependent SU(2) force induced by spin-orbit coupling is tuned in magnitude against the constant (but opposite) topological force. At a particular value of $\boldsymbol{E}$ field the total transverse force is zero, thus achieving a complete cancellation of spin transverse separation.

In conclusion, we have designed a spin transverse separation system similar to the spin Hall that is easy to realize and detect. We analyzed the forces acting on a spin-polarized current in a 2DEG heterostructure device in the presence of external vertical $\boldsymbol{B}$ fields with net chirality and Rashba SO interactions within the 2DEG. The spin-up electrons will experience a transverse force due to the non-Abelian nature of the SU(2) gauge field arising from the SO effects and the topological gauge fields arising from the chirality of the external $\boldsymbol{B}$ field. The advantage of the device is that the essentially perpendicular external $\boldsymbol{B}$ field induces a vertically aligned spin current which can experience the full effects of the SU(2) and $U(1) \otimes U(1)$ gauge fields. This removes the need for very short and clean device, simplifying experimental effort to implement. Additionally, the tunability of the magnitude and direction of the SU(2) gauge field enables either a gate-voltage-induced enhancement or cancellation of the spin transverse separation arising from the constant topological field, thus enabling the device to turn "on" and "off" spin separation. It is worth noting that the tunability of the SU(2) gauge field is based on the physics of intrinsic but not extrinsic spin separation. This device could thus be used to determine the type of spin transverse separation which is predominant in the 2DEG system.

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